Mathematically Modeling Motion of Cells in Porous Media

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Fifth Annual MIT-PRIMES Conference

May 16, 2015

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Modeling cell-motion in porous media

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• Motion of particles in simple systems

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- Motion of particles in simple systems
 - Random free-fluid motion

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- Motion of particles in simple systems
 - Random free-fluid motion
 - Random one-dimensional lattice walks

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- Motion of particles in simple systems
 - Random free-fluid motion
 - Random one-dimensional lattice walks
- Properties of the truly random infinite walk in \mathbb{Z}^d

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• More realistic medium

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- More realistic medium
- Study boundaries

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- More realistic medium
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- Find parameters that effect asymptotic behavior

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- More realistic medium
- Study boundaries
- Find parameters that effect asymptotic behavior
- Compare analytics to simulations and experiment

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 \bullet Walk in ${\mathbb Z}$ without wall

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- \bullet Walk in ${\mathbb Z}$ without wall
 - *t* = number of steps in one run
 - *r* = number of runs

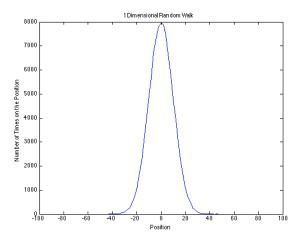
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• Walk in \mathbb{Z} without wall: $p_{\leftarrow} = 0.5$, t = 100, $r = 10^5$

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• Walk in $\mathbb Z$ without wall: $p_{\leftarrow}=0.5,\;t=100,\;r=10^5$



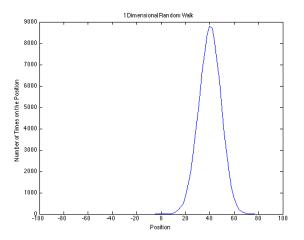
A (10) × (10) × (10)

• Walk in $\mathbb Z$ without wall: $p_{\leftarrow}=0.3,\;t=100\;r=10^5$

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• Walk in \mathbb{Z} without wall: $p_{\leftarrow} = 0.3$, $t = 100 \ r = 10^5$



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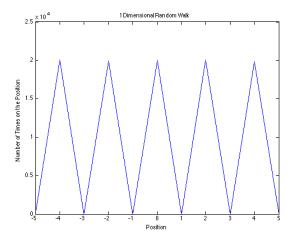
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• Walk in \mathbb{Z} with wall: $p_{\leftarrow} = 0.5$, t = 100, $r = 10^5$, n = 5

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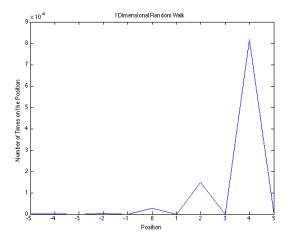
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• Walk in \mathbb{Z} with wall: $p_{\leftarrow} = 0.5$, t = 100, $r = 10^5$, n = 5



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• Walk in $\mathbb Z$ with wall: $p_{\leftarrow} = 0.3$, $t = 10^5$, $r = 10^4$



• Represent the porous medium as an array of points

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- Represent the porous medium as an array of points
- Convert the lattice grid to a graph

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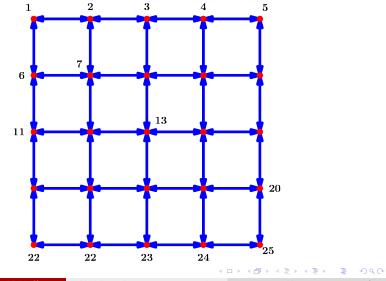
- Represent the porous medium as an array of points
- Convert the lattice grid to a graph
- Assign probabilities to the edges

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- Represent the porous medium as an array of points
- Convert the lattice grid to a graph
- Assign probabilities to the edges
- Run the walk and graph a histogram

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• Walk in \mathbb{Z}^2 with a negligible wall

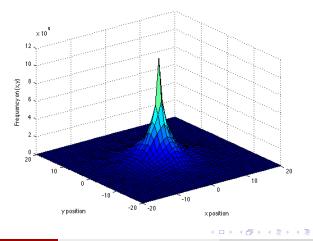
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• Walk in \mathbb{Z}^2 with a negligible wall: persistency 0, t = 100, r = 1000, and n = 20.

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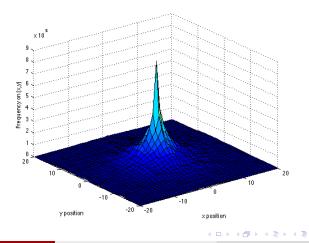
• Walk in \mathbb{Z}^2 with a negligible wall: persistency 0, t = 100, r = 1000, and n = 20.



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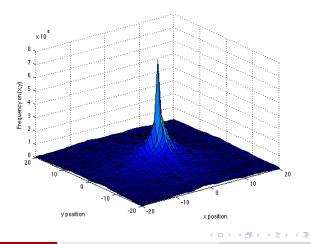
• Walk in \mathbb{Z}^2 with a negligible wall: persistency 0.5, t = 100, r = 1000, and n = 20.



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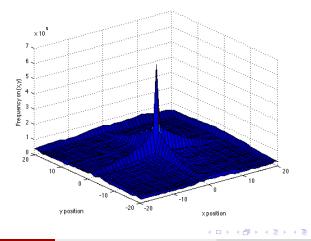
• Walk in \mathbb{Z}^2 with a negligible wall: persistency 0.7, t = 100, r = 1000, and n = 20.



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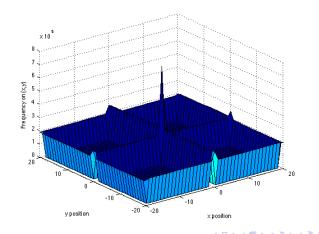
• Walk in \mathbb{Z}^2 with a negligible wall: persistency 0.9, t = 100, r = 1000, and n = 20.



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• Walk in \mathbb{Z}^2 with a negligible wall: persistency 0.999, t = 100, r = 1000, and n = 20.



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Super-Gaussian

• Gaussian curve:
$$f(x) = \exp\left(-\left|\frac{x}{a}\right|^2\right)$$

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Super-Gaussian

• Gaussian curve:
$$f(x) = \exp\left(-\left|\frac{x}{a}\right|^2\right)$$

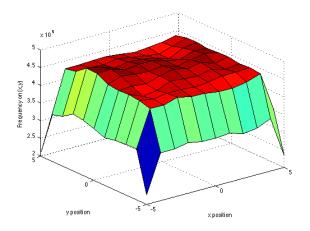
• Super-gaussian curve: $f(x) = \exp\left(-\left|\frac{x}{a}\right|^n\right)$, for n < 2

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 \bullet Walk in \mathbb{Z}^2 with wall

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• Walk in \mathbb{Z}^2 with wall: persistency 0, n = 5, $t = 10^4$, r = 100.

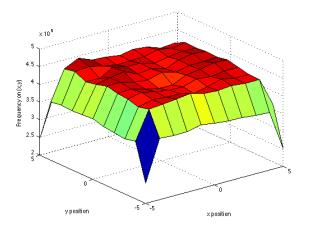


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• Walk in \mathbb{Z}^2 with wall: persistency 0.2, n = 5, $t = 10^4$, r = 100.

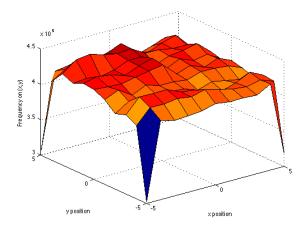


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• Walk in \mathbb{Z}^2 with wall: persistency 0.4, n = 5, $t = 10^4$, r = 100.



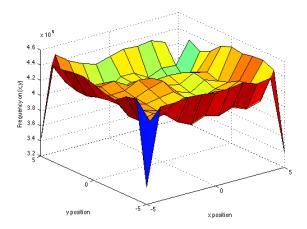
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Extra dimension

• Walk in \mathbb{Z}^2 with wall: persistency 0.5, n = 5, $t = 10^4$, r = 100.

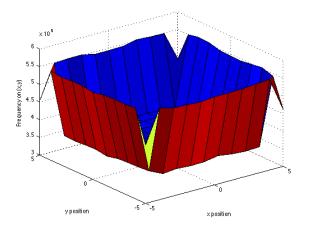


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Extra dimension

• Walk in \mathbb{Z}^2 with wall: persistency 0.7, n = 5, $t = 10^4$, r = 100.



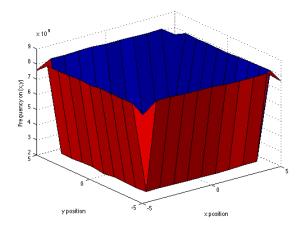
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Extra dimension

• Walk in \mathbb{Z}^2 with wall: persistency 0.9, n = 5, $t = 10^4$, r = 100.



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Observations

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• Simulations match existing theories

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Observations

- Simulations match existing theories
- Memory favors boundaries, asymptotically

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• Generalized model

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- Generalized model
- Finding a small set of determining parameters

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Generalized model

• Finding a small set of determining parameters

• More study on boundaries

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- Generalized model
- Finding a small set of determining parameters
 - More study on boundaries
- Match with experiment
 - Can we use Markov chains?
 - How do simulated mixing times compare with experiment?

Definition

The mixing time of a Markov chain ${\cal M}$ is the time until ${\cal M}$ tends to settle to a steady state.

- Generalized model
- Finding a small set of determining parameters
 - More study on boundaries
- Match with experiment
 - Can we use Markov chains?
 - How do simulated mixing times compare with experiment?

Definition

The mixing time of a Markov chain ${\cal M}$ is the time until ${\cal M}$ tends to settle to a steady state.

• Deriving diffusion coefficients (Fick)

- Generalized model
- Finding a small set of determining parameters
 - More study on boundaries
- Match with experiment
 - Can we use Markov chains?
 - How do simulated mixing times compare with experiment?

Definition

The mixing time of a Markov chain ${\cal M}$ is the time until ${\cal M}$ tends to settle to a steady state.

- Deriving diffusion coefficients (Fick)
- Lazy random walks

Acknowledgements

A huge thanks to:

- Prof. Jörn Dunkel
- Mentor Andrew Rzeznik
- MIT-PRIMES
- My parents

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